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1 The Mouse Constraint (and its Jacobian)

If one let x be a rigid body position in world space then it can define the "mouse" constraint such that

$$C = (x + r) - x_{mouse} = 0, \tag{1.1}$$

geometrically stating that the difference between the rigid body particle in world frame (at location r in body frame from Eq. (1.1)) and the mouse position in world frame (x_{mouse}) should be zero.

Treating x_{mouse} as constant and differentiating Eq. (1.1) with respect to time the velocity constraint is written as

$$\dot{C} = J \begin{pmatrix} v \\ \omega \end{pmatrix} = v + \omega \times r = 0, \tag{1.2}$$

geometrically stating that the linear and angular relative velocity at r should be zero; v and ω are the linear and angular velociy of the body, respectively.

Considering a single body simulation, one can let this constraint second body mass goes to infinite (practically, a zero mass), and write the Jacobian as

$$J \in \mathbb{R}^{3\times 6} = \begin{pmatrix} 1 & 0 & 0 & 0 & -r_z & r_y \\ 0 & 1 & 0 & r_z & 0 & -r_x \\ 0 & 0 & 1 & -r_y & r_x & 0 \end{pmatrix}, \tag{1.3}$$

instead of defining it as $J \in \mathbb{R}^{12\times 6}$. Therefore, from Eq. (1.2),

$$JV = J \begin{pmatrix} v \\ \omega \end{pmatrix} = 0. \tag{1.4}$$

Note that this Jacobian can be easely extended to be attached to a multiple body solver. However, the rest of this text describes how to compute the effective mass $JM^{-1}J^T$ assuming $J \in \mathbb{R}^{3\times 6}$ in order to calculate the corrective impulse

$$\lambda = (JM^{-1}J^T)^{-1} \left(-JV + \frac{\delta}{\Delta t} C \right), \tag{1.5}$$

where δ is the *Baumgarte* factor and Δt the time-step.

1.1 Calculating $\boldsymbol{JM^{-1}J^T}$

$$J \in \mathbb{R}^{3\times 6} = \begin{pmatrix} 1 & 0 & 0 & 0 & -r_z & r_y \\ 0 & 1 & 0 & r_z & 0 & -r_x \\ 0 & 0 & 1 & -r_y & r_x & 0 \end{pmatrix}. \tag{1.6}$$

$$J^T \in \mathbb{R}^{6\times3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & r_z & -r_y \\ -r_z & 0 & r_x \\ r_y & -r_x & 0 \end{pmatrix}. \tag{1.7}$$

$$M^{-1} \in \mathbb{R}^{6\times6} = \begin{pmatrix} m^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & m^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & m^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & {I^{-1}}_{xx} & {I^{-1}}_{yx} & {I^{-1}}_{zx} \\ 0 & 0 & 0 & {I^{-1}}_{xy} & {I^{-1}}_{yy} & {I^{-1}}_{zy} \\ 0 & 0 & 0 & {I^{-1}}_{xz} & {I^{-1}}_{yz} & {I^{-1}}_{zz} \end{pmatrix}. \tag{1.8}$$

$$JM^{-1} \in \mathbb{R}^{3\times6} = \begin{pmatrix} m^{-1} & 0 & 0 & -{I^{-1}}_{xy}r_z + {I^{-1}}_{xz}r_y & -{I^{-1}}_{yy}r_z + {I^{-1}}_{yz}r_y & -{I^{-1}}_{zy}r_z + {I^{-1}}_{zz}r_y \\ 0 & m^{-1} & 0 & -{I^{-1}}_{xz}r_x + {I^{-1}}_{xx}r_z & -{I^{-1}}_{yz}r_x + {I^{-1}}_{yx}r_z & -{I^{-1}}_{zz}r_x + {I^{-1}}_{zx}r_z \\ 0 & 0 & m^{-1} & {I^{-1}}_{xy}r_x - {I^{-1}}_{xx}r_y & {I^{-1}}_{yy}r_x - {I^{-1}}_{yx}r_y & {I^{-1}}_{zy}r_x - {I^{-1}}_{zx}r_y \end{pmatrix}. \tag{1.9}$$

$$JM^{-1}J^T \in \mathbb{R}^{3\times3} = \begin{pmatrix} m^{-1} + \left({I^{-1}}_{yy}{r_z}^2 + \left(-{I^{-1}}_{yz} - {I^{-1}}_{zy}\right)r_y r_z + {I^{-1}}_{zz}{r_y}^2\right) & \left({I^{-1}}_{zy}r_z - {I^{-1}}_{zz}r_y\right)r_x + \left(-{r_z}^2{I^{-1}}_{xy} + r_y{I^{-1}}_{xz}r_z\right) & \left(-{I^{-1}}_{yy}r_z + {I^{-1}}_{yz}r_y\right)r_x + \left(r_y r_z{I^{-1}}_{xy} - {r_y}^2{I^{-1}}_{xz}\right) \\ \left({I^{-1}}_{yz}r_z - {I^{-1}}_{zz}r_y\right)r_x + \left(-{I^{-1}}_{yx}{r_z}^2 + {I^{-1}}_{zx}r_y r_z\right) & m^{-1} + {I^{-1}}_{zz}{r_x}^2 + \left(-{I^{-1}}_{xz} - {I^{-1}}_{zx}\right)r_z r_x + {I^{-1}}_{xx}{r_z}^2 & -{I^{-1}}_{yz}{r_x}^2 + \left({I^{-1}}_{yx}r_z + r_y{I^{-1}}_{xz}\right)r_x - {I^{-1}}_{xx}r_y r_z \\ \left(-{I^{-1}}_{yy}r_z + {I^{-1}}_{zy}r_y\right)r_x + \left({I^{-1}}_{yx}r_y r_z - {I^{-1}}_{zx}{r_y}^2\right) & -{I^{-1}}_{zy}{r_x}^2 + \left(r_z{I^{-1}}_{xy} + {I^{-1}}_{zx}r_y\right)r_x - {I^{-1}}_{xx}r_y r_z & m^{-1} + {I^{-1}}_{yy}{r_x}^2 + \left(-r_y{I^{-1}}_{xy} - {I^{-1}}_{yx}r_y\right)r_x + {I^{-1}}_{xx}{r_y}^2 \end{pmatrix} \Rightarrow$$
$$\begin{pmatrix} m^{-1} & 0 & 0 \\ 0 & m^{-1} & 0 \\ 0 & 0 & m^{-1} \end{pmatrix} + \begin{pmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{pmatrix} \begin{pmatrix} {I^{-1}}_{xx} & {I^{-1}}_{yx} & {I^{-1}}_{zx} \\ {I^{-1}}_{xy} & {I^{-1}}_{yy} & {I^{-1}}_{zy} \\ {I^{-1}}_{xz} & {I^{-1}}_{yz} & {I^{-1}}_{zz} \end{pmatrix} \begin{pmatrix} 0 & r_z & -r_y \\ -r_z & 0 & r_x \\ r_y & -r_x & 0 \end{pmatrix}. \tag{1.10}$$